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The Black-Litterman Model in Central Bank Practice: Study for Turkish Central Bank

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ABSTRACT

The Modern Portfolio Theory is based on Markowitz Mean-Variance portfolio optimization. The Black-Litterman Model uses a Bayesian approach which combines expert's views about assets involved in optimization with equilibrium returns implied by market capitalization weights, and as a result we get expected returns which can be put in Mean-Variance optimization. After the global financial crisis 2007-2009 emerging countries' central banks started to restructure their international reserves. During the crisis gold outperformed other assets by 42% and thus explicitly demonstrated its feature as safe haven asset. Therefore, including gold into the investment portfolio helps to survive economic turbulence with less harm. However, the question what percentage of portfolio should be allocated to gold to avoid the above mentioned problem remains unanswered. In this paper using the Black-Litterman model we consider this problem in case of the Central Bank of the Republic of Turkey.

Keywords: Black-Litterman model; Modern Portfolio Theory; Expected return.

1. Introduction

The recent financial crisis of 2008 resulted in paradigm shift among the investors. Collapse of institutions which believed to be "to big to fail" and downgrading of US government bonds for the first time since their inception push investors to reconsider the allocation of their investment portfolios. Given the record level of international reserves accumulated by emerging countries' central banks (CBs) rose a paramount question; how to allocate the reserves in most profitable way. CBs known as the most risk-averse investors have three main objectives such as: (i) safety, (ii) liquidity and (iii) profitability. Therefore, managing of foreign reserves should meet all three objectives accordingly. The utmost priority of a CB is safety i.e. the preservation of the capital value of reserves, which leads to investments with low volatility and subsequently low returns. Following the Asian currency crisis of 1998 Asian countries tremendously increased their US dollar reserves. For example, FinancialTimes (2014) states that China alone holds 3.95 trillion US dollar denominated assets in its reserves. US treasuries believed to meet all criteria of safe asset therefore constituting 67% of total foreign reserves IMF2011 (2011). However, according to Litterman (2007), in long term US treasuries are very risky assets therefore, reserve managers advised to diversify their portfolios away from US dollar denominated assets. The second objective is liquidity according to which CBs hold their portfolios in liquid assets to ensure immediate intervention in case of potential economic turbulence. And last objective is profitability, where reserve managers seek for the desirable level of return on reserve assets. Therefore, yet central banks aim to keep reserves safe and liquid simultaneously they are seeking for high returns to fulfill efficient management of increasing reserves Zhang et al. (2012). Making right decisions in asset allocation which cohere with above mentioned three main objectives is utmost priority of reserve managers (Morahan and Mulder, 2013).

To shed a light into the central banker's attitude aftermath the crisis Morahan and Mulder (2013) conducted a survey for IMF amongst central bankers of 160 member nations. Their findings suggest that most reserve managers would consider holding a higher proportion of safer assets, and would also contemplate applying more robust frameworks. To ensure that at least part of foreign reserves consists of hedge or safe haven assets is an important factor which determines the performance of CB and therefore of a country during financial turmoil. Baur and Lucey (2009) suggest that hedge asset is an asset which outperforms average portfolio during the normal times of economy, while safe haven asset is an asset which outperforms the average portfolio during the extreme times only.

2. Literature Review

The main idea of the Modern Portfolio Theory (MPT) is a diversification of assets. By including assets which are uncorrelated to each other the investor is likely to minimize the risk while maximizing the profit. Markowitz (1952) explained it as follows: "If security returns were not correlated, diversification could eliminate risk. It would be like flipping a large number of coins: we cannot predict with confidence the outcome of a single flip; but if a great many coins are flipped we can virtually be sure that heads appear on approximately one-half of them. To reduce risk it is necessary to avoid a portfolio whose securities are all highly correlated with each other." (Markowitz (1952), p.5). MPT model helps the investor to build a portfolio which considers the level of risk acceptable for the investor. Before forming the portfolio the manager should analyse each asset to be included separately. Thus, the manager can determine for each asset: i) the expected return, ii) indicators of the risk, iii) relationship with other assets. After isolated analyses of the asset one can calculate various weight combinations to determine the profitability with given risk. To determine the optimal portfolio first of all the investor should find the best feasible set of relation between expected return and risk, which lies on the efficient frontier where all possible efficient portfolios take place and any portfolio below the efficient frontier has either higher risk or lower expected return (Markowitz, 1959). However, there are numbers of criticisms addressed to MV optimization. Practitioners find the model inconvenient due to requirements like having an opinion on each involved asset and providing a precise number for expected returns. Furthermore, there are complains that even negligible changes in expected returns may result in significant reallocation in portfolio which leads to a less satisfactory portfolios (Michaud, 1989).

Christadoulakis (2002) argues that when the level of a risk-aversion is relatively higher too much weight is given to an asset with a lower volatility rather than diversifying a portfolio into a larger number of assets with low correlation. Theoretical limitations of the MV framework were described in ref. Brandt (2009). According to Brandt (2009), the framework has three limitations such as i) as expected utility maximization the framework implies quadratic utility which leads to a preference specification problem; ii) higher-order returns are not considered due to second-order approximation of expected utility maximization; iii) MV framework oversee real world investment problems. According to Fernandes et al. (2012), the MV framework is not suitable for reserve managers, since the latter have multiple goals and are subject to behavioural biases such as risk-aversion, asymmetric risk-taking behaviour. One of the most prominent problems related to the MV approach famously known as a substitution problem, is that when dealing with two assets let us say a and b with the same level of risk and where the expected return of asset a is slightly higher than the expected return of the asset b the model would allocate all weight to the asset a which obviously not rationale from investors perspective (Best and Grauer (1991), Fernandes et al. (2012)Morahan and Mulder (2013), Brandt (2009), Avramov and Guofu (2010), Wachter (2010)). Fisher Black and Robert Litterman of Goldman Sachs introduced Black Litterman model in their well-known paper Black and Litterman (1991). The model is an improved version of Markowitz's MV approach. B-L model helps reserve manager to overcome main limitations of MV optimization such as input sensitivity of a portfolio, lack of intuition and high concentration. A new expected return vector is derived by implementing Bayesian paradigm which combines unique views of a reserve manager regarding to the expected returns of a single asset (absolute view) or multiple assets (relative view) with market equilibrium vector. Therefore, used along with an allocation process BL model derives more balanced and more diversified portfolios. Meanwhile, Markowitz (1952) explained in details the implementation of the model thus making an application reproducible, the BL model being a sophisticated approach is not easy to reproduce, unlike MV paradigm. The first literature explained step by step implementation of the B-L model is Idzorek (2004). The prior literature for example, Black and Litterman (1991) do not provide an important details and also data essential to replicate the process. The parameter introduced in the paper is a fraction

$$(P'\Omega^{-1}P)[(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1} \tag{1}$$

which represents weights of investor's views. In their subsequent paper in the model, Black and Litterman (1992), presents more detailed discussion by providing input data and numbers of results. Even though, vital equations necessary to implement the model are provided the posterior equation, which is:

$$P\Sigma P' = \sum_{i=1}^{k} \frac{1}{k} \frac{(1/LC_i * CF)}{\tau} \tag{2}$$

was not given and introduced only in ref. Black and Litterman (1999). Idzorek (2004) takes its place in the BL model literature as a step by step guide into the reproduction of the model, with numerical examples and detailed explanations.

3. Main Sections

Black and Litterman (1991) were motivated to further develop the Mean-Variance approach, due to the shortcomings encountered while implementing Markowitz's model in practical portfolio management added the fact that the approach hasn't had a crucial impact in practice. Black and Litterman (1992) tried to derive improved portfolio models by introducing a new approach in predicting expected returns. Meanwhile, the portfolio required to be at the efficient frontier. If this condition is oversaw then by using Mean-Variance approach it is possible to derive a "better" portfolio. Therefore, it should be noted that the Black-Litterman model is not a novelty but the improved version of the Markowitz's model in terms of calculating expected returns. As a result the portfolios derived using Black-Litterman model differs substantially from those generated using Markowitz's model. Figure 1 illustrates the scheme of the Black-Litterman model.

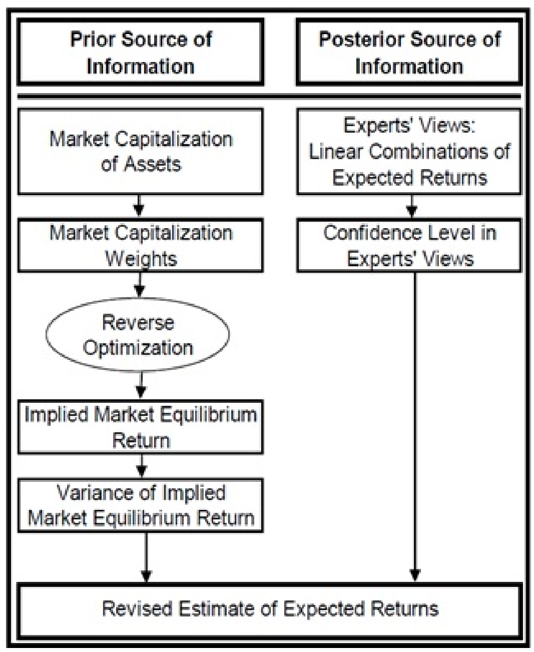

Figure 1: The Black-Litterman Model

As a neutral starting point the Black-Litterman Model uses "equilibrium" returns. Equilibrium returns are calculated using either the CAPM (an equilibrium pricing model) or a reverse optimization method in which the vector of implied expected equilibrium returns (Π) is extracted from known information. Using matrix algebra, one solves for Π in the formula,

$$\Pi = \delta \Sigma w, \tag{3}$$

where:
δ - risk-aversion coefficient;

$\Sigma - n \times n$ covariance matrix of excess returns;
$w - n \times 1$ column vector of market capitalization weights of assets.

The risk-aversion coefficient δ represents the level of risk the investor willing to take for obtaining higher return. Higher δ characterizes unwillingness of an investor in taking risk, while small δ represents investor's willingness in taking risk for higher return. Next we apply the step where Bayes' law is applied to combine investors' views with market equilibrium returns (Π). The posterior Combined Return Vector E[R] is derived as follows:

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right], \tag{4}$$

where:
E[R] - New Combined Return Vector ($n \times 1$ column vector)
τ - scalar
Σ - Covariance Matrix of Returns ($n \times n$ matrix)
P - Identifies the assets involved in the views ($k \times n$ matrix or $1 \times n$ row vector in the special case of 1 view)
Ω - Diagonal covariance matrix of error terms in expressed views representing the level of confidence in each view ($k \times k$ matrix)
Π - Implied Equilibrium Return Vector ($n \times 1$ column vector)
Q - View Vector ($k \times 1$ column vector).

Note: ($'$) indicates the transpose and (-1) indicates the inverse of corresponding matrix, k represents the number of views and n expresses the number of assets (Idzorek, 2004). Two parameters in equation (2) which do not have a straightforward definition and have to be defined by practitioner are τ and Ω. In this study, we set τ to 1. Ω is derived using the following formula:

$$\Omega = \tau P \Sigma P' \tag{5}$$

4. Data and Implementation

In this study empirical application is based on 5 indices of bonds, equities and commodity. We employ 10-year US bond index, 10-year EURO bond. For equities, S&P 500 and STOXX 600 are used as the proxies for US and Eurozone equities, respectively. Monthly total return indices are used over the sample period from January 2010 to December 2012, with a total of 36 observations. All total return indices are calculated in a log-return style based on a US-dollar denomination and the 3-month US T-Bill is taken as the risk-free rate. Return and volatility of gold are based on the London pm fix rate benchmark, published by the London Bullion Market Association (LBMA). Table 1 reports the descriptive statistics for all asset classes considered.

Table 1: Descriptive Statistics

Name	Market	Type	Mean	Standard Deviation
US Bonds	USA	Long-term Bonds	2.57 %	0.74 %
EURO Bonds	Euro Zone	Long-term Bonds	3.58 %	0.56 %
US Equity	USA	Equity	0.91 %	4.42 %
Euro Equity	Euro Zone	Equity	3.58 %	4.67%
Gold	—	Commodity	1.25 %	3.66 %

Current allocation of reserves of CBRT is illustrated in Figure 2.

As April 2014 CBRT's reserves are amounted to 129,732.66 billion USD. Where, 75% consists of securities, 16% consists of gold bullion, 8% currency and deposits and the rest is allocated for SDR and IMF reserve position. Black-Litterman model is applied to derive new portfolio. First we derive Variance-Covariance Matrix (Σ) as presented in Table 2.

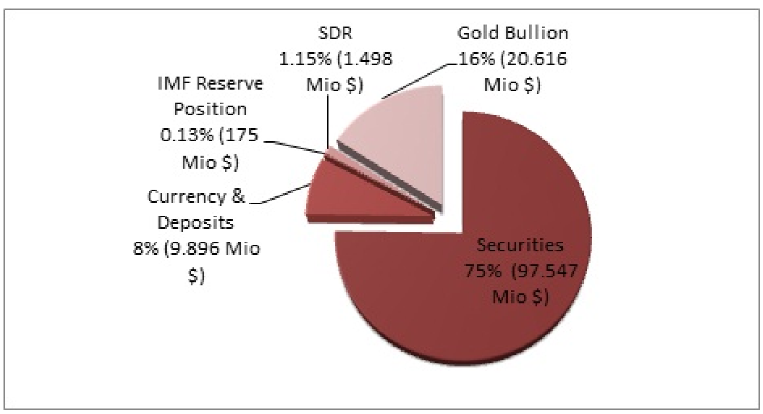

Figure 2: Current allocation of reserves of CBRT

Using Equation 3 we calculate equilibrium excess returns on given assets and based on equilibrium excess return reserve managers deliver their unique views. We assume that following the global financial crises of 2008 official investors favour safe assets for investment purposes. The views reserve managers formulate with respect to the equilibrium excess returns are as follows: (1) Gold will outperform US bonds by 2%; and (2) STOXX600 will outperform S&P500 by 1.5%. The confidence level for both given views is 50%. Market weights, implied equilibrium returns and BL expected returns are illustrated in Table 3.

Table 2: Variance-Covariance Matrix of Excess Returns (Σ)

	S&P500	STOXX600	US Bonds	EU Bonds	GLD
S&P500	0.001984	-0.00139	-0.0002	0.000124	0.000266
STOXX600	-0.00139	0.002643	0.000238	-0.00038	-0.00056
US Bonds	-0.0002	0.000238	0.001649	0.000305	0.000451
EU Bonds	0.000124	-0.00038	0.000305	0.00073	0.000861
GLD	0.000266	-0.00056	0.000451	0.000861	0.0013

According to Table 3, the difference between expected returns derived from B-L approach and implied equilibrium return for US Bonds is -0.96%. The reason is that the investor has a negative view on the future performance of US Bonds. There is no significant fluctuation in Euro Bonds with expected returns 3.49% and 3.10% and Black-Litterman expected returns and implied equilibrium returns respectively. Negative views on US Equity resulted in negative expected return which is -0.13% for B-L approach, 0.23% less than implied equilibrium returns. Positive intuitions of the investors related to the future performance of gold derived the highest expected return on B-L approach which is 4.79%, explicitly outperforming other assets included into the portfolio.

Table 3: Return Vectors and Resulting Portfolio Weights

Asset Class	New Combined Return Vector $E[R]$	Implied Equilibrium Return Vector Π	Difference $E[R]-\Pi$	New Weight w_{BI}	Market Capitalization Weight w_{mkt}
US Bonds	4.34%	5.30%	-0.96%	15.12%	29.58%
EU Bonds	3.49%	3.10%	0.39%	22.00%	28.54%
US Equity	-0.13%	0.10%	-0.23%	6.92%	10.64%
EU Equity	1.07%	1.00%	0.07%	15.96%	15.24%
Gold	4.79%	4.20%	0.59%	40.00%	16.00%
			Sum	100%	100%

Indeed, the performance of gold after the global financial crisis of 2008, and gradually diminishing trust in the financial future of current reserve currencies such as US Dollar and EURO push official investors of emerging countries towards safe haven assets, and gold seems to be the best alternative for diversification.

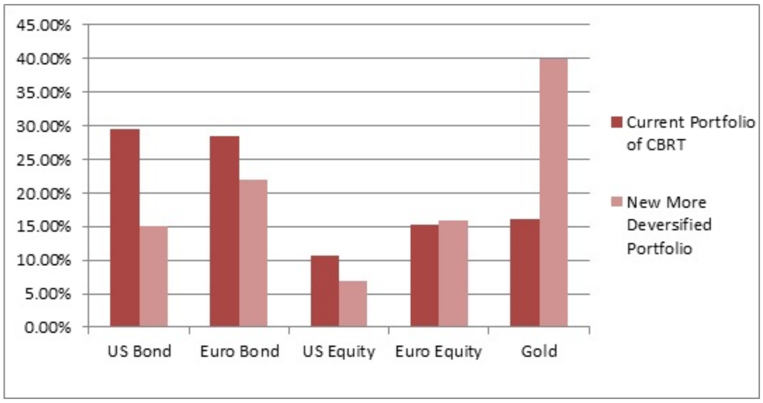

Figure 3: Portfolio allocations (Current vs Posterior)

Figure 3 illustrates the asset allocation of current portfolio of CBRT, where majority of reserves is allocated to US Bonds and Euro Bonds 29.58% and 28.54% respectively. Whereas, portfolio in a portfolio allocated according to the Black-Litterman expected returns the share of US Bonds is decreased almost two times to 15.12%, following with a sharp decrease in US Equity which decreased from 10.64% to 6.92%. There are slight changes in Euro Bonds and Euro Equity. While, most interesting allocation was in favour of gold, the share of the latest increased from 16% to 40%. It is explicit that our results are consistent with the views of the investors.

5. Conclusion

Current changes in economy drive investors in general and central bankers in particular to create a portfolio which is able to perform relatively good during the times of financial distress. However, one of the main objectives of official investors is to ensure the safety of reserves. Therefore, reserve managers restricted in investing into risky assets. Yet, return is welcomed. By applying the Black-Litterman approach we have derived new portfolio for CBRT. Our posterior portfolio has higher expected return E[R] with relatively smaller standard deviation (σ). Intuition of official investors incorporated into the posterior expected return vector to derive a portfolio which is supposed to perform relatively better during the financial turbulence.

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